Temporal aggregation of equity return time-series models

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Abstract

With large volatility observed in stock markets around the world over the last few years, many actuaries are now being urged to employ stochastic models to measure the solvency risk generated from insurance products with equity-linked guarantees. There are a large number of potential stochastic models for equity returns. Insurance regulators, both in Europe and North America, normally do not restrict the use of any stochastic model that reasonably fits the historical baseline data. However, in the U.S. and Canada, the final model must be calibrated to some specified distribution percentiles. The emphasis of the calibration process remains on the tails of the equity return distribution over different holding periods. In this paper, we examine the effect of temporal aggregation on classes of stochastic equity return models that are commonly used in actuarial practice. The advantages of choosing a closed (under temporal aggregation) class of processes for modelling asset returns and equity-linked guarantees are discussed. Actuarial applications of temporal aggregation using S&P500 data are given.

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1. Introduction

With large volatility observed in stock markets around the world over the last 3 years (see Table 1), many actuaries are now being asked to employ stochastic models to measure the solvency risk generated from insurance products with equity-linked guarantees.

The movement toward stochastic scenario modelling is consistent with a trend among insurance regulators. The U.K.’s Financial Services Authority (FSA) is restructuring its capital requirement framework. After 2004, all British life insurers are subject to individual capital adequacy standards, requiring an insurer to assess the adequacy of its capital resources using its own stochastic scenario testing. Although Continental Europe still uses formula-driven minimum solvency capital requirements that do not take into account the specific asset side, a review of these standards is under way, aimed at aligning required capital more closely with a company’s actual risks [20]. The March 2002 final report of the Canadian Institute of Actuaries (CIA) Task Force on Segregated Fund Investment Guarantees [27] provides useful guidance for appointed actuaries applying stochastic techniques to value segregated fund guarantees in a Canadian GAAP valuation environment. Monthly TSE 300 total return indices from January 1956 to December 1999 are recommended as the baseline series for Canada. In the United States, the Life Capital Adequacy Subcommittee (LCAS) of the American Academy of Actuaries (AAA) issued the C-3 Phase II Risk-Based Capital (RBC) proposal in

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December 2002. The full report of the Subcommittee [19] was published by the American Academy of Actuaries and is available from the Academy’s website. Monthly S&P 500 total return indices from January 1945 to October 2002 are used as the baseline series.

Even though there is no mandatory class of stochastic models for fitting the baseline data, it is recommended that the final model be calibrated to some specified distribution percentiles in the U.S. and Canada. The calibration process emphasises tails of the equity return distribution over three different holding periods: 1 year, 5 years and 10 years [14,16]. The CIA report [27] recommends a set of model calibration points based on extrapolation. The extrapolation is performed by fitting three models of stock returns using the monthly data. Then, empirical distributions are simulated with these models for the 1-year, 5-year and 10-year accumulation factors. The models used are: a regime switching log-normal model with two regimes, a stochastic volatility log-normal model and a stable distribution model. The extrapolation results are employed to develop a set of 11 calibration points—3 percentile values for 1, 5 and 10-year time horizons, plus ranges for annual mean and standard deviation. In the United States, the AAA adopts an approach similar to the CIA, where a regime-switching log-normal model is fitted to monthly total return data. The extrapolation results are then used to develop a set of 30 accumulated wealth calibration points that must be materially met by the equity model used to determine capital requirements. Like the CIA requirements, these points are at 1, 5 and 10-year time horizons. Only 15 of the 30 points usually apply (corresponding to one of the tails of the distribution), depending upon the insurance product under evaluation [19,20].

Both the CIA and AAA recognise that empirical data are too sparse to provide credible calibration criteria for the annual, 5-year and 10-year points. Yet, for long-term investors such as insurance companies and pension funds, the dynamic of the long-term accumulation factor is very important. In this paper, instead of totally relying on the empirical extrapolation approach adopted by the CIA and AAA, we examine the theoretical linkage between a short term (monthly) stochastic model and its corresponding stochastic model for longer time horizon.

Let \( S_t \) be the monthly total return index value at \( t, \) for \( t = 0, 1, 2, \ldots, n \). Define

\[
    r_t = \log \left( \frac{S_t}{S_{t-1}} \right)
\]

as the \( log \) \( return \) for the \( t \)\( ^{th} \) month. The log return series for a \( m \)-month non-overlapping holding period can be constructed by

\[
    R_T = \log \left( \frac{S_{mT}}{S_{m(T-1)}} \right) = \sum_{t=m(T-1)+1}^{mT} (r_t)
\]

for \( T = 1, 2, \ldots, N \), and we assume that \( N = \lfloor n/m \rfloor \) is an integer. The accumulation factor for the period \( [m(T - 1), mT) \) (in months) is given by

\[
    A_T = \left( \frac{S_{mT}}{S_{m(T-1)}} \right) = \exp(R_T).
\]

Eq. (2) is called temporal aggregation in the time series econometric literature. The parameter \( m \) is termed the order of aggregation. Thus, we can set \( m = 12, 60 \) and 120 to obtain the 1-year, 5-year and 10-year accumulation factors from the disaggregated \( r_t \) series.
In recent years there has been growing interest in studying the effect of temporal aggregation on financial asset returns (e.g., see [7,21]). In general, we say that a model is closed under temporal aggregation if the model keeps the same structure, with possibly different parameter values, for any data frequency. This property has also been discussed in the actuarial literature [15,17].

The rest of this paper is organised as follows. In Section 2, we examine the effect of temporal aggregation on classes of stochastic equity return models that are commonly used in actuarial practice. The advantages of choosing a closed (under temporal aggregation) class of processes for modelling asset returns are discussed. Section 3 illustrates the application of temporal aggregation results by some actuarial examples. Concluding remarks are given in Section 4.

2. Temporal aggregation

In this section, temporal aggregation of commonly used equity return models is studied. It should be noted that if the monthly stochastic equity return model is closed under temporal aggregation, the parameters of the monthly model can be directly calibrated to generate longer term accumulation factors.

2.1. The independent log-normal (ILN) model

We first consider the traditional log-normal equity return model. The log-normal model has a long and illustrious history, and has become “the workhorse of the financial asset pricing literature” [5]. The log-normal model assumes that log returns are independently and identically distributed (IID) normal variates with a constant mean and a constant variance.

It is well-known that the ILN model is closed under temporal aggregation (or scale invariance in the finance literature). If the monthly log return \( r_t \) as defined in Eq. (1) follows a normal distribution, that is

\[
 r_t \sim N(\mu, \sigma^2),
\]

then the aggregated log return series for a \( m \)-month non-overlapping holding period also follows a normal distribution, i.e.,

\[
 R_T \sim N(m\mu, m\sigma^2).
\]

The corresponding accumulation factor variable \( A_T \) as defined in Eq. (2) therefore follows the log-normal distribution.

2.2. The independent log-stable (ILS) model

Stable distributions are a class of probability laws that have intriguing theoretical and practical properties. The class is characterised by [18] in his study of the sums of independent identically distributed variables. The application of stable laws to actuarial/financial modelling follows from the fact that stable distributions generalise the normal (Gaussian) distribution to accommodate heavy tails and skewness, which are frequently seen in investment data (e.g., see [8,10,23]).

There are a number of possible parameterisations of the class of stable distributions. Nolan [24] lists more than 10 different definitions of stable parameters. In this paper, we employ the “S0” parameterisation, which is better suited to numerical calculations than other representations [22]. Under this definition, the class of stable distributions is described by four parameters, which we call \((\alpha, \beta, \gamma, \delta)\). A random variable \( Y \) is S0(\( \alpha, \beta, \gamma, \delta \)), if its characteristic function takes the form:

\[
 \Psi(t) = E[\exp(itY)] = \begin{cases} 
 \exp \left( -\gamma |t|^\alpha \left[ 1 + i\beta \text{sign}(t) \left( \tan \left( \frac{\pi \alpha}{2} \right) \left( |t|^{1-\alpha} - 1 \right) \right) \right] + i\delta t \right), & \alpha \neq 1, \\
 \exp \left( -\gamma |t| \left[ 1 + i\beta \text{sign}(t) \left( \frac{2}{\pi} \left( \ln(|t|) \right) \right) \right] + i\delta t \right), & \alpha = 1.
\end{cases}
\]

It should be noted that Gaussian distributions are special cases of stable laws with \( \alpha = 2 \) and \( \beta = 0 \); more precisely, \( N(\mu, \sigma^2) = S0(2, 0, \sigma/\sqrt{2}, \mu) \).

The reason for terming this class of distributions stable is that they retain their main distributional characteristics under addition [24]. This means that the stable model is closed under temporal aggregation. If the monthly log return
$r_i$ are independently and identically distributed as stable, that is
\[ r_i \sim S0(\alpha, \beta, \gamma, \delta), \]  
(6)
then the aggregated log return series for a $m$-month non-overlapping holding period also follows a stable distribution, i.e.,
\[ R_T \sim S0(\alpha^*, \beta^*, \gamma^*, \delta^*). \]  
(7)
The aggregated and disaggregated stable parameters are related by
\[ \begin{align*}
\alpha^* &= \alpha, \\
\beta^* &= \beta, \\
\gamma^* &= m^{1/\alpha} \gamma, \\
\delta^* &= \begin{cases} 
  m\delta + m^{1/\alpha} \beta \gamma \tan \left( \frac{\pi \alpha}{2} \right), & \alpha \neq 1, \\
  m\delta + 2\pi m^{1/\alpha} \beta \gamma \ln(m^{1/\alpha} \gamma), & \alpha = 1.
\end{cases}
\end{align*} \]  
(8)
The corresponding accumulation factor variable $A_T$ therefore follows the log-stable distribution.

2.3. The linear ARMA model

Wilkie [31,32] developed linear stochastic asset models for United Kingdom data. Wilkie’s model is based on the orthodox Box and Jenkins’ ARMA (autoregressive moving average) modelling techniques [4]. Suppose that time series $Y_t$ has the stationary and invertible ARMA($p$, $q$) representation
\[ \phi(L)Y_t = \theta(L)\alpha_t, \]  
(9)
where $L$ is the backshift operator such that $L^s Y_t = Y_{t-s}$.
\[ \begin{align*}
\phi(L) &= 1 - \phi_1 L - \cdots - \phi_p L^p, \\
\theta(L) &= \theta_1 L - \cdots - \theta_q L^q,
\end{align*} \]
$\phi(L)$ and $\theta(L)$ have all of their roots outside of the unit circle, and $\alpha_t$ is Gaussian white noise with zero mean and constant variance $\sigma_\alpha^2 < \infty$. Without a loss of generality, we assume that $E[Y_t] = 0$ (should $E[Y_t] = \mu_Y \neq 0$ then, instead of working with $[Y_t]$, the mean-corrected process $\{\tilde{Y}_t = Y_t - \mu_Y\}$ will be used). For all integers $s$, defining the autocovariance $\gamma(s)$ at lag $s$ by $\text{Cov}[Y_t, Y_{t-s}]$, we have
\[ \gamma(s) = E[Y_t, Y_{t-s}]. \]
The stochastic process in (9) can be characterised by
\[ \Theta = (\sigma_\alpha^2, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q). \]  
(10)
Equivalently, it can be fully represented by its autocovariances, $\Gamma = \{\gamma(0), \gamma(1), \ldots, \gamma(p+q)\}$, i.e., precisely the same information as in $\Theta$ is contained in $\Gamma$.

In general, the class of linear ARMA processes is closed under temporal aggregation. If the disaggregated series follows an ARMA($p$, $q$) model, then the temporal aggregated series follows an ARMA($c$, $d$) process where the aggregated orders ($c$, $d$) could be the same as or different from the original orders ($p$, $q$). The parameters and the orders of the aggregated model can be derived through the relationship of autocovariances between the disaggregated and aggregated series (see [30] and references therein).

The first-order autoregressive process is often employed to model the first-lag serial correlation that is observed in many stock return data. As an example, we illustrate the results discussed above using an AR(1) model. Assume that the monthly log return $r_t$ follows an AR(1) process,
\[ r_t = \phi r_{t-1} + \alpha_t, \quad \alpha_t \sim N(0, \sigma_\alpha^2). \]
First, we derive the lag-s autocovariance function of the m-period aggregated log return variable,

\[
\text{Cov}[R_T, R_{T+s}] = \begin{cases} 
[m + 2(m - 1)\phi + 2(m - 2)\phi^2 + \ldots + 2\phi^{m-1}] \frac{\sigma^2}{1 - \phi^2}, & \text{if } s = 0, \\
[1 + \phi + \phi^2 + \ldots + \phi^{m-1}]^2 \frac{\phi^{m(s-1)+1}}{1 - \phi^2} \sigma^2, & \text{if } s = \pm 1, \pm 2, \ldots 
\end{cases} 
\]

Eq. (11) implies that \( R_T \) follows an ARMA(1, 1) process, i.e.,

\[(1 - \phi^s L)R_T = (1 - \theta^s L)a_t^*, \quad a_t^* \sim N(0, \sigma^2_{\varepsilon_t}).\]

with

\[\phi^* = \phi^m,\]

and \(|\theta^*| < 1\) is the solution of the following quadratic equation

\[
\frac{(\phi^m - \theta^m)(1 - \phi^m \theta^m)}{1 - 2\phi^m \theta^* + \theta^2} = \frac{\phi(1 + \phi + \phi^2 + \ldots + \phi^{m-1})^2}{m + 2(m - 1)\phi + 2(m - 2)\phi^2 + \ldots + 2\phi^{m-1}}.
\]

Finally, \(\sigma^2_{\varepsilon_t}\) can be computed from the \(\text{Var}[R_T]\) equation in (11).

2.4. The GARCH model

Time-varying volatility models have been popular since the early 1990s in financial research and applications, following the influential papers by [2,9]. Stochastic models of this type are known as generalised autoregressive conditional heteroscedastic (GARCH) models in the time series econometrics literature. GARCH processes are useful because these models are able to capture empirical regularities of asset returns such as thick tails of unconditional distributions, volatility clustering and negative correlation between lagged returns and conditional variance [11,29].

Let \(a_t = (r_t - \mu)\) be the mean-corrected log return. Then, \(a_t\) follows a GARCH\((p, q)\) model if

\[
\epsilon_t = \frac{a_t}{\sqrt{h_t}}, \quad h_t = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j},
\]

where \(\{\epsilon_t\}\) is a sequence of IID random variables with mean zero and unit variance, \(\omega > 0, \alpha_i \geq 0, \beta_j \geq 0,\) and \(\sum_{k=1}^{\max(p,q)} (\alpha_k + \beta_k) < 1\) with \(\alpha_k = 0\) for \(k > p\) and \(\beta_k = 0\) for \(k > q\).

Little is known about the temporal aggregation on a general GARCH\((p, q)\) process. For some lower order GARCH models, Drost and Nijman [7] show that they are closed under temporal aggregation. Fortunately, in most practical applications, lower order GARCH processes are adequate for modeling equity return data. In this section, we consider a GARCH\((1, 1)\) process and its innovation following a Student \(t\) distribution: i.e., \(\epsilon_t\) in Eq. (12) has a marginal \(t\) distribution with mean zero, unit variance and degrees of freedom \(\nu\), and the conditional variance has the following representation:

\[h_t = \omega + \beta h_{t-1} + \alpha a_{t-1}^2,\]

and the unconditional kurtosis of \(a_t\) is \(\kappa\). Following [7], we find that the aggregated return for a \(m\)-month non-overlapping period can be “weakly” approximated by a GARCH\((1, 1)\) process with the corresponding parameters:

\[
\mu^* = m \mu, \quad \omega^* = m \omega \left\{ \frac{1 - (\alpha + \beta)^m}{1 - (\alpha + \beta)} \right\}, \quad \alpha^* = (\alpha + \beta)^m - \beta^*,
\]

\[
\kappa^* = 3 + \frac{(\kappa - 3)}{m} + 6(\kappa - 1) \left\{ \frac{m - 1 - m(\alpha + \beta) + (\alpha + \beta)^m}{m2(1 - \alpha - \beta)^2(1 - 2\alpha \beta - \beta^2)} \right\},
\]

and \(|\beta^*| < 1\) is the solution of the following quadratic equation:

\[
\frac{\beta^*}{1 + \beta^*} = \frac{\Theta[(\alpha + \beta)^m] - \Lambda}{\Theta[1 + (\alpha + \beta)^2m] - 2\Lambda},
\]
with
\[
\Theta = m(1 - \beta)^2 + \left\{ \frac{2m(m - 1)(1 - \alpha - \beta)^2(1 - 2\alpha\beta - \beta^2)}{(\kappa - 1)(1 - (\alpha + \beta)^2)} \right\}
+ 4 \left\{ \frac{m - 1 - m(\alpha + \beta) + (\alpha + \beta)^m[(\alpha - \alpha\beta(\alpha + \beta) - 1 - (\alpha + \beta)^2]}{(1 - (\alpha + \beta)^2} \right\},
\]
and
\[
\Lambda = \frac{[\alpha - \alpha\beta(\alpha + \beta)][1 - (\alpha + \beta)]^{2m}}{1 - (\alpha + \beta)^2}.
\]

The degrees of freedom of the marginal $t$ distribution ($\nu^*$) for the aggregated GARCH model can be derived via the aggregated kurtosis, $\kappa^*$ [1].

### 2.5. The RSLN model

In recent years, the use of regime switching log-normal (RSLN) processes for modelling maturity guarantees has been gaining popularity. Hardy [14] proposes using Markov-type regime switching log-normal (RSLN) processes for modelling monthly equity returns. The RSLN model is defined as

\[
rt = \mu S_t + \sigma S_t \varepsilon_t,
\]
where $S_t = 1, 2, \ldots, k$ denotes the unobservable state indicator, which follows an ergodic $k$-state Markov process, and $\varepsilon_t$ is a standard normal random variable that is IID over time. In most situations, $k = 2$ or 3 (i.e., two- or three-regime models) is sufficient for modelling monthly equity returns [14]. The stochastic transition probabilities that determine the evolution in $S_t$ are given by

\[
\Pr[S_{t+1} = j|S_t = i] = p_{ij},
\]

\[
0 < p_{ij} < 1, \quad \sum_{j=1}^{k} p_{ij} = 1 \quad \text{for all } i,
\]
so that the states follow a homogenous Markov chain.

Research into the temporal aggregation of RSLN models is scanty and very much in its infancy. As a starting point to study the effect of temporal aggregation on RSLN processes, we examine the limiting behaviour (i.e., $m \rightarrow \infty$) of a simple two-regime RSLN model. Let

\[
rt = \begin{cases} 
\varepsilon_t^{(1)} \sim N(\mu_1, \sigma_1^2), \\
\varepsilon_t^{(2)} \sim N(\mu_2, \sigma_2^2),
\end{cases}
\]

with transition probability matrix

\[
P = \begin{pmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{pmatrix}, \quad 0 < p_{ij} < 1.
\]

This implies that the vector of steady-state (ergodic) probabilities is

\[
\left( \begin{array}{c} 
\pi_1 \\
\pi_2
\end{array} \right) = \begin{pmatrix}
p_{21} \\
p_{12} + p_{21}
\end{pmatrix}.
\]
It is well-known that the limiting structure of time series aggregates from a covariance stationary process is white noise (e.g., see [12,26,30]). Timmermann [28] gives the autocovariance function of the RSLN model in (14),

$$
\gamma(s) = \pi_1\pi_2(\mu_1 - \mu_2)^2\text{vec}(\mathcal{P}^s)\eta.
$$

(16)

where \(\text{vec}(\cdot)\) is the vector of a matrix operator [13], and \(\eta = (\pi_2, -\pi_2, -\pi_1, \pi_1)'\). Cox and Miller [6] derive that

$$
\mathcal{P}^s = \begin{pmatrix}
\pi_1 & \pi_2 \\
\pi_1 & \pi_2
\end{pmatrix} + (1 - p_{12} - p_{21})^s
\begin{pmatrix}
\pi_2 & -\pi_2 \\
-\pi_1 & \pi_1
\end{pmatrix}.
$$

(17)

Combining (16) and (17), we have

$$
\gamma(s) = \pi_1\pi_2(\mu_1 - \mu_2)^2(1 - p_{12} - p_{21})^s.
$$

This autocovariance function is obviously geometrically bounded because \(|1 - p_{12} - p_{21}| < 1\), which is guaranteed by the restriction in (15) that all elements of \(\mathcal{P}\) be positive. Hence, the limiting structure for the temporal aggregates of a two-regime RSLN model is white noise.

Deriving the effect of temporal aggregation on a more general (i.e., more than two regimes and finite values of \(m\)) RSLN model is not a trivial task. Research in this direction is in process.

3. Applications

Quantile matching is the key of the Life Capital Adequacy Subcommittee (LCAS) calibration requirement in the United States [19]. In this application, we consider the monthly S&P 500 total return series from January 1945 to October 2002, which is the baseline series recommended by the Subcommittee. The main focus of this example is to numerically illustrate the quantile results using model aggregation formulae derived in the previous section. These results are then compared to the published calibration requirements. It is not our objective to recommend the “best” fitted model for the S&P 500 series.

From the five classes of models discussed in the last section, we find that the classes of ILS models and GARCH models are reasonably fitted to the data. First, we consider the class of independent stable distributions. Assume that the monthly S&P 500 log return follows an IID stable distribution, i.e.,

$$
r_t \sim \text{S0}(\alpha, \beta, \gamma, \delta),
$$

see Eqs. (5)–(8) in Section 2. Methods of computing the maximum likelihood estimation (MLE) of stable parameters are discussed by [24,25] provides software for estimating and analysing stable models. The fitted stable parameters for the monthly series \(r_t\) are given in Table 2. The implied parameters for the aggregated stable models with different orders of aggregation \(m\) are calculated using Eq. (8). The results are listed in Table 2. The corresponding accumulation factor \((A_T)\) for a holding period \(m\), as defined in Eq. (2), follows an ILS model. The ILS model is simple and tractable. The cumulative distribution function (CDF) and the probability distribution function (PDF) of \(A_T\) are easy to be evaluated [25].

Financial asset return time series usually exhibit a characteristic known as volatility clustering, in which large changes tend to follow large changes, and small changes tend to follow small changes. Furthermore, probability distributions for asset returns often exhibit fatter tails than the standard normal distribution. The fat tail phenomenon

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Monthly model</th>
<th>Aggregated model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(m = 12)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1.8678</td>
<td>1.8678</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.7591</td>
<td>-0.7591</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.0273</td>
<td>0.1032</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.0128</td>
<td>0.1705</td>
</tr>
</tbody>
</table>
is known as excess kurtosis in the finance literature. It is well-known that the GARCH model can accommodate these unique features (heavy tails and volatility clustering), which occur frequently in observed stock returns [3].

The GARCH(1, 1) model with marginal \( t \) distribution is fitted to the S&P 500 monthly total return series. The fitted GARCH parameters and their corresponding implied parameters for the aggregated models with different orders of aggregation \( m \) are given in Table 3. The implied parameters are computed using formulae discussed in Section 2.4.

Finally, lower quantiles for the 1-year accumulation factor using the aggregated ILS and GARCH models are obtained. Table 4 compares these quantiles to the calibration criteria derived from the S&P 500 empirical data by the AAA [19,20]. The GARCH quantiles closely match to the AAA calibration points, but the ILS quantiles are too large as compared to the criteria.

### 4. Conclusion

This paper examines the effect of temporal aggregation on five classes of stochastic equity return models that are commonly used in actuarial practice. Analytical formulae for the linkages between some aggregated and disaggregated stochastic models are presented. If a model is closed under temporal aggregation, the parameters of the lower frequency model can be directly implied by the higher frequency (i.e., more data) model. This property is particularly useful when we need to study distributions and dynamics of longer term (for instance, more than 10 years) accumulation factors for equity-linked insurance products. The data on 10-year (or longer) accumulation factors is often very limited, even for the mature markets in Europe and North America.

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