Final Exam

Student Name: ____________________

Course: Calculus II
Instructor: Qun Mo
Time: 8:00–10:00
Date: April 28, 2010

Total Scores: 100 points

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1. (20pts) Determine whether the statement is true or false. You do not need to give your reasons.

(a) (4pts) If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum a_n \) is convergent.

(b) (4pts) If \( a_n > 0 \) and \( \sum a_n \) converges, then \( \sum (-1)^n a_n \) converges.

(c) (4pts) For any vectors \( \mathbf{u} \) and \( \mathbf{v} \) in \( V_3 \), \( (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0 \).

(d) (4pts) If \( \mathbf{u}(t) \) and \( \mathbf{v}(t) \) are differentiable vector functions, then
\[
\frac{d}{dt} [\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}'(t)
\]

(e) (4pts) \( f_{xy} = \frac{\partial^2 f}{\partial x \partial y} \).

2. (40pts) Calculation.

(a) (8pts) Determine whether the series \( \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1} \) is convergent or divergent.

(b) (8pts) Find the radius of convergence and interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(x + 2)^n}{n4^n} \).
(c) (8pts) Find the Maclaurin series for $f$ and its radius of convergence. You may use either the direct method (definition of a Maclaurin series) or known series such as geometric series, binomial series, or the Maclaurin series for $e^x$, $\sin(x)$, and $\tan^{-1} x$.

$$f(x) = \frac{x^2}{1 + x}$$

(d) (8pts) Use series to evaluate the following limit.

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

(e) (8pts) Find all second partial derivatives of $f$.

$$f(x, y) = 4x^3 - xy^2$$
3. (10pts) Find the acute angle between two diagonals of a cube.

4. (10pts) For the curve given by $\mathbf{r}(t) = \langle t^3/3, t^2/2, t \rangle$, find (a) the unit tangent vector, (b) the unit normal vector, and (c) the curvature.
5. (10pts) If \( u = x^y \), show that
\[
\frac{x \partial u}{y \partial x} + \frac{1}{\ln x} \frac{\partial u}{\partial y} = 2u
\]

6. (10pts) Use power series to solve the initial-value problem
\[
y'' + xy' + y = 0 \quad y(0) = 0 \quad y'(0) = 1
\]