Symposium on Number Theory and Representation Theory

会议手册
（Information）
Symposium on Number Theory and Representation Theory

May 14 - May 16, 2021 Zhejiang University
Hangzhou, China

Invited Speakers:
Kei Yuen Chan  Shanghai Center for Mathematical Sciences
Sarah Dijols  Yau Mathematical Sciences Center
Bingrong Huang  Shandong University
Ning Li  Beijing International Center for Mathematical Research
Wen Wei Li  Beijing International Center for Mathematical Research
Jiajun Ma  Shanghai Jiao Tong University
Zhifeng Peng  Soochow University
Qinghua Pi  Shandong University, Weihai
Xiaolei Wan  Beijing International Center for Mathematical Research
Liuquan Wang  Wuhan University
Ping Xi  Xi’an Jiaotong University
Bin Xu  Yau Mathematical Sciences Center
Jun Yu  Beijing International Center for Mathematical Research

Organizers:
刘东文(maliu@zju.edu.cn)、齐治(zhi.qi@zju.edu.cn)、高帆(gaofan@zju.edu.cn)

Time:
9:00-11:40am, 2:00-5:40pm, May 14-May 16

Venue:
Sir Run Run Shaw Science Building 211, Yuquan Campus, Zhejiang University,
Hangzhou (浙江大学邵逸夫科学馆211)
### Meeting on 14-16 May 2021

(Starting From the afternoon of May 13, the participants can check in at the reservation hotel)

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Title and Abstract

Time: 9:00-11:40am, 2:00-5:40pm, May 14-May 16
Venue: Sir Run Run Shaw Science Building 211, Yuquan Campus, Zhejiang University

Speaker: Bingrong Huang（黄炳荣）
Affiliation: Shandong University
Title: Quantum variance for automorphic forms
Abstract: In this talk, we will present results on the quantum variances for Eisenstein series and for dihedral Maass forms on modular surfaces. The resulting quadratic forms are compared with the classical variance (Ratner) and the quantum variance for cusp forms (Luo-Sarnak). They coincide after inserting certain subtle arithmetic factors, including the central values of certain L-functions. (Based on joint work with Stephen Lester.)

Speaker: Ning Li（李宁）
Affiliation: Beijing International Center for Mathematical Research
Title: Degenerate principal series representations and nilpotent invariants
Abstract: In this talk, we will discuss two geometric invariants: wave front cycles and associated cycles, attached to irreducible constituents of degenerate principal series representations of $Sp(2n, R)$. The main strategy is to combine the explicit description of these constituents obtained by Lee and Zhu and the machinery of “theta lift” of nilpotent orbits via moment maps.

Speaker: Bin Xu（徐斌）
Affiliation: Yau Mathematical Sciences Center
Title: Functoriality of endoscopic transfer for general symplectic and even orthogonal groups
Abstract: Langlands’ functoriality conjecture reveals a deep connection of automorphic representations among different reductive groups. Most known cases of functoriality fit into the theory of endoscopy, which concerns a group $G$ and its endoscopic groups. We study the endoscopic theory for $G$ being a quasisplit general symplectic or even orthogonal group over a number field, and prove the functoriality of endoscopic transfer for tempered automorphic representations of these groups under some technical assumption.
**Speaker:** Wen Wei Li (李文威)

**Affiliation:** Beijing International Center for Mathematical Research

**Title:** Full stable trace formula for the group $Mp(2n)$

**Abstract:** The metaplectic covering $Mp(2n)$ of $Sp(2n)$ plays a pivotal role in various aspects of representation theory and arithmetics. In order to harness the full power of Arthur’s methods in this setting, we need a stable trace formula for $Mp(2n)$. Thus far, only the elliptic terms have been stabilized. In this talk, I will report an ongoing work on the full stabilization, which will hopefully grant access to the whole genuine discrete automorphic spectrum of $Mp(2n)$, for general $n$. This is based on the techniques of Arthur and Moeglin-Waldspurger, who worked with linear reductive groups or their twisted analogues.

**Speaker:** Jun Yu (余君)

**Affiliation:** Beijing International Center for Mathematical Research

**Title:** Restriction of unitary representations of Spin(N,1) to parabolic subgroups

**Abstract:** The orbit method predicts a relation between restrictions of irreducible unitary representations and projections of corresponding coadjoint orbits. In this talk we will discuss branching laws for unitary representations of Spin(N,1) restricted to parabolic subgroups and the corresponding orbit geometry. In particular, we confirm Duflo’s conjecture in this setting. This is a joint work with Gang Liu (Lorraine) and Yoshiki Oshima (Osaka).

**Speaker:** Qinghua Pi (皮庆华)

**Affiliation:** Shandong University, Weihai

**Title:** Some results on automorphic forms of cubic level

**Abstract:** With the methods of the relative trace formula and the classification of simple supercuspidal representations, we establish some Fourier formulas for automorphic new forms for $GL_2$ of cubic level. As applications, we prove some results on the non-vanishing of modular $L$-values, the weighted Weyl’s law and the bias of root numbers. This talk is based on the joint works with Yingnan Wang, Lei Zhang and Zhi Qi.

**Speaker:** Sarah Dijols

**Affiliation:** Yau Mathematical Sciences Center

**Title:** The completed $L$-function for $G_2$

**Abstract:** This talk is based on a joint work (recently submitted) where we obtain the functional equation of the completed $L$ function for modular forms on $G_2$. The first part of the talk will cover the Rankin-Selberg method, and explain the adaptations needed to treat the case of $G_2$. 

Speaker: Liuquan Wang (王六权)
Affiliation: Wuhan University
Title: Representations of mock theta functions
Abstract: Motivated by the works of Liu, we provide a unified approach to find Appell-Lerch series and Hecke-type series representations for mock theta functions. We establish a number of parameterized identities with two parameters $a$ and $b$. Specializing the choices of $(a, b)$, we not only give various known and new representations for the mock theta functions of orders $2, 3, 5, 6$ and $8$, but also present many other interesting identities. We find that some mock theta functions of different orders are related to each other, in the sense that their representations can be deduced from the same $(a, b)$-parameterized identity. Furthermore, we introduce the concept of false Appell-Lerch series. We then express the Appell-Lerch series, false Appell-Lerch series and Hecke-type series in this work using the building blocks $m(x, q, z)$ and $f_{a,b,c}(x, y, q)$ introduced by Hickerson and Mortenson, as well as $\tilde{m}(x, q, z)$ and $\tilde{f}_{a,b,c}(x, y, q)$ introduced in this paper. We also show the equivalences of our new representations for several mock theta functions and the known representations. This talk is based on a joint work with Dandan Chen.

Speaker: Kei Yuen Chan (陈佳源)
Affiliation: Shanghai Center for Mathematical Sciences
Title: Ext-vanishing phenomenon in branching laws of classical groups
Abstract: Ext-vanishing is useful in the study of cohomology of representations. A classical example of Ext-vanishing is that there are no higher extensions between two discrete series of a reductive groups over local fields. In the context of branching laws of classical groups, D. Prasad predicts higher Ext-vanishing between tempered representations (or more generally generic representations). In this talk, I shall explain various examples of Ext-vanishing, including conjectures, old and new results. Results are centered around general linear groups, in which a main tool-- left-right derivatives will also be explained if time permits.

Speaker: Jiajun Ma (马家骏)
Affiliation: Shanghai Jiao Tong University
Title: Special unipotent representations of real classical groups and theta correspondence.
Abstract: Special unipotent representations of a real reductive group are certain irreducible admissible representations attached to the nilpotent orbits of its dual group. Barbasch and Vogan established the theory of special unipotent representations for complex reductive groups (construction, unitarizability, etc.) in the '80s. In this talk, I will discuss the recent joint work with Dan Barbasch, Binyong Sun, and Chengbo Zhu on the classification and unitarity of special unipotent representations of the real classical groups (real symplectic groups, real orthogonal groups, and the metaplectic groups).
**Speaker:** Zhifeng Peng (彭志峰)  
**Affiliation:** Soochow University  
**Title:** The representation classification of the exceptional group $G_2$  
**Abstract:**  
In 2002, Assume the fundamental lemma, Arthur given a stabilization of the trace formula by the endoscopic theory.  
In 2008, Ngo proved the fundamental lemma.  
In 2013, Arthur classified the automorphic representations of classic groups by the twisted trace formula of $GL(n)$ and the standard trace formula of classic groups, when the twisted trace formula holds.  
In 2014, Waldspurger built the twisted trace formula for the reductive groups. Our goal is to give the endoscopic classification of automorphic representations of exceptional group $G_2$ by the twisted trace formula of $PGSO(8)$, and the standard trace formula of $G_2$.  
In this talk, we will introduce the basic objects and background.

**Speaker:** Ping Xi (郗平)  
**Affiliation:** Xi'an Jiaotong University  
**Title:** Lang--Trotter conjecture for CM elliptic curves  
**Abstract:** For any elliptic curve $E$ over $Q$ and any non-zero integer $r$, the Lang--Trotter conjecture has predicted the asymptotic behaviours of the number of good primes $p \leq x$, denoted by $\Pi_{E,r}(x)$, such that the Frobenius trace of $E$ at $p$ is equal to the given integer $r$. Quite recently, we are able to prove an estimate for $\Pi_{E,r}(x)$ which confirms the upper bound part of the conjecture for CM elliptic curves. Moreover, intimate connections of this conjecture and Hardy--Littlewood conjecture can also be established to characterize the shape of the Lang--Trotter constant in $\Pi_{E,r}(x)$. This is based on the joint work with Daqing Wan (in progress).

**Speaker:** Xiaolei Wan (万小磊)  
**Affiliation:** Beijing International Center for Mathematical Research  
**Title:** Sakellaridis--Venkatesh conjecture and theta correspondence  
**Abstract:** In this talk, I will introduce the Sakellaridis-Venkatesh conjecture on the decomposition of global period, and give examples related to this conjecture. More specifically, the case $X = U(2) \backslash SO(5)$. I will determine the Plancherel decompositions of $L^2(X,\nu)$, where $\nu$ is a local place. Then I will prove the local relative character identity. In the global setting, I will give the factorization of the global period of $X = U(2) \backslash SO(5)$, where the local functional comes from the local Plancherel decomposition. The example $X = U(2) \backslash SO(5)$ is slightly beyond the SV conjecture but we still have a decomposition of the global period as the sum of two factorizable elements.
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