2017 Number Theory Days in Hangzhou

Schedule

Saturday, Sep 23

Sheraton Grand Hangzhou Wetland Park Resort

- 8:30: Opening ceremony
- 8:45: Aleksandar Ivić: On some results and problems involving Hardy's function $Z(t)$
- 9:35: Hourong Qin: Congruent numbers, quadratic forms and K_2
- 10:15: Break
- 10:35: Jia-Yan Yao: Fractional hypergeometric functions for function fields and weak transcendence in positive characteristic
- 11:20: Guangshi Lü: On sum of one prime, two squares of primes and powers of 2
- 12:00: Lunch
- **Problem Session**
- 14:00: Zhi-Wei Sun: Some conjectures on representations of positive rational numbers
- 14:25: Liuquan Wang: Modular forms and k-colored generalized Frobenius partitions
- 14:50: Jin-Hui Fang: Additive complements of the squares
- 15:15: Victor Zhenyu Guo: Consecutive primes and Beatty sequences
- 15:40: Banquet dinner at Longjing Village

Sunday, Sep 24

Brook Hotel Hangzhou

- 8:45: Preda Mihăilescu: The Kummer-Vandiver conjecture
- 9:35: Zhi-Wei Sun: Further results on Hilbert's tenth problem
- 10:15: Break
- 10:35: Shaofang Hong: The Igusa's local zeta functions of super elliptic curves
- 11:20: Zhi-Hong Sun: A kind of orthogonal polynomials and related identities

Abstract

Saturday, Sep 23

On some results and problems involving Hardy's function $Z(t)$

Aleksandar Ivić
Serbian Academy of Arts and Sciences, Belgrade

Hardy's classical function $Z(t)$ is defined, for real t, as

$$Z(t) := \zeta(\frac{1}{2}+it)\chi(\frac{1}{2}+it)^{-1/2}, \zeta(s) = \chi(s)\zeta(1-s),$$

This talk will cover some problems involving Hardy's function, including moments at points where $|Z(t)|$ is maximal, large values of Hardy's function, and distribution of positive and negative values of $Z(t)$.

Congruent numbers, quadratic forms and K_2

Hourong Qin
Department of Mathematics, Nanjing University, Nanjing

A celebrated theorem due to Tunnell gives a criterion for a positive integer to be congruent (under the BSD). We show that if a square-free and odd (respectively, even) positive integer n is a congruent number, then

$$\#\{(x,y,z) \in \mathbb{Z}^3 | n = x^2 + 2y^2 + 32z^2\} = \#\{(x,y,z) \in \mathbb{Z}^3 | n = 2x^2 + 4y^2 + 9z^2 - 4yz\},$$

respectively,

$$\#\{(x,y,z) \in \mathbb{Z}^3 | \frac{n}{2} = x^2 + 4y^2 + 32z^2\} = \#\{(x,y,z) \in \mathbb{Z}^3 | \frac{n}{2} = 4x^2 + 4y^2 + 9z^2 - 4yz\}.$$

If we assume that the weak Brich-Swinnerton-Dyer conjecture is true for the elliptic curves $E_n : y^2 = x^3 - n^2x$, then, conversely, these equalities imply that n is a congruent number.

Some applications are given. In particular, we show that if $p \equiv 1 \pmod 8$ is a congruent number, then the 8-rank of $K_2 O_{\mathbb{Q}(\sqrt{p})} = 1$.

Fractional hypergeometric functions for function fields and weak transcendence in positive characteristic

Jia-Yan Yao
Department of Mathematics, Tsinghua University, Beijing

In this talk we shall present a weak transcendence criterion in positive characteristic, and apply it to study fractional hypergeometric functions for function fields, introduced by D. S. Thakur. For these functions, it is known when they are transcendental functions or not, and in the case that they are entire transcendental functions, we shall apply the above criterion to show the weak transcendence of their special values at nonzero algebraic arguments.

On sum of one prime, two squares of primes and powers of 2

Guangshi Lü
School of Mathematics, Shandong University, Jinan

In this talk, I shall introduce Linnik's almost Goldbach problem, which states that each large even integer is a sum of two primes and a bounded number of powers of 2, and its analogue that every large odd integer N can be written as a sum of one prime, two squares of primes and a bounded number of powers of 2. It is proved that every sufficiently large odd integer can be written as a sum of one prime, two squares of primes and 17 powers of 2.

Sunday, Sep 24

The Kummer-Vandiver conjecture

Preda Mihăilescu
Mathematisches Institut, Georg-August-University, Goettingen

The talk will cover historical aspects from the genesis of this conjecture to its present significance, and, time allowing, will give some ideas about its proof.

Further results on Hilbert's tenth problem

Zhi-Wei Sun

Department of Mathematics, Nanjing University, Nanjing

Hilbert's Tenth Problem (HTP) asks for an effective algorithm to test whether an arbitrary polynomial equation

$$P(x_1, \cdots, x_n) = 0$$

(with integer coefficients) has solutions over the ring $\mathbb{Z}$ of the integers. This was finally solved by Matiyasevich in 1970 negatively.

In this talk we introduce the speaker's further results on HTP. We present a sketch of the proof of the speaker's main result that there is no effective algorithm to determine whether an arbitrary polynomial equation $P(x_1, \dots, x_{11}) = 0$ (with integer coefficients) in 11 unknowns has integral solutions or not. We will also mention some other results of the speaker, for example, there is no algorithm to test for any $P(z_1, \dots, z_{17}) \in \mathbb{Z}[z_1, \dots, z_{17}]$ whether $P(z_1^2, \dots, z_{17}^2) = 0$ has integral solutions, and also there is a polynomial $Q(z_1, \dots, z_{20}) \in \mathbb{Z}[z_1, \dots, z_{20}]$ such that

$$\{Q(z_1^2, \dots, z_{20}^2) : z_1, \dots, z_{20} \in \mathbb{Z}\} \cap \{0, 1, 2, \dots\}$$

coincides with the set of all primes.

The Igusa's local zeta functions of super elliptic curves

Shaofang Hong

Mathematical College, Sichuan University, Chengdu

In this talk, we speak about the local zeta functions for two variables polynomial g, where $g(x, y) = 0$ is the super elliptic curve with coefficients in a non-archimedean local field of positive characteristic. We prove the rationality of these local zeta functions and describe explicitly all their candidate poles. This is a joint work with Qiuyu Yin.

A kind of orthogonal polynomials and related identities

Zhi-Hong Sun

School of Mathematical Sciences, Huaiyin Normal University, Huaian

In this talk we introduce the polynomials $\{d_n^{(r)}(x)\}$ and $\{D_n^{(r)}(x)\}$ given by $d_n^{(r)}(x) = \sum_{k=0}^{n} \binom{x+r+k}{k}\binom{x-r}{n-k} (n \geq 0)$, $D_0^{(r)}(x) = 1, D_1^{(r)}(x) = x$ and $D_{n+1}^{(r)}(x) = xD_n^{(r)}(x) - n(n+2r)D_{n-1}^{(r)}(x) (n \geq 1)$. We show that $\{D_n^{(r)}(x)\}$ are orthogonal polynomials for $r > -\frac{1}{2}$, and establish many identities for $\{d_n^{(r)}(x)\}$ and $\{D_n^{(r)}(x)\}$, especially obtain a formula for $d_n^{(r)}(x)^2$ and the linearization formulas for $d_m^{(r)}(x)d_n^{(r)}(x)$ and $D_m^{(r)}(x)D_n^{(r)}(x)$. As an application we extend recent work of Sun and Guo.

Problem Session

Saturday, Sep 23

Some conjectures on representations of positive rational numbers

Zhi-Wei Sun
Department of Mathematics, Nanjing University, Nanjing

In this talk we introduce various conjectures of the speaker on representations of positive rational numbers. For example, we conjecture that any positive rational number can be written as the sum of finitely many distinct unit fractions whose denominators have the form $p-1$ (or $p+1$) with p prime. Another typical conjecture states that any positive rational number can be written as m/n with m and n positive integers such that the sum of the m-th prime and the n-th prime is a square. We will provide some motivations and numerical data supporting the conjectures.

Modular forms and k-colored generalized Frobenius partitions

Liuquan Wang
Division of Mathematical Sciences, Nanyang Technological University, Singapore

Let k and n be positive integers. Let $c\phi_k(n)$ denote the number of k-colored generalized Frobenius partitions of n and $C\Phi_k(q)$ be the generating function of $c\phi_k(n)$. In this talk, we first introduce a new way for finding representations of $C\Phi_k(q)$ by using the theory of modular forms. In particular, we find alternative representations of $C\Phi_k(q)$ for $k \leq 17$. Second, we present some relations between $c\phi_k(n)$ and the ordinary partition function $p(n)$. Lastly, an interesting congruence modulo prime powers satisfied by $c\phi_k(n)$ will be given.

This is a joint work with Professors Heng Huat Chan and Yifan Yang.

Additive complements of the squares

Jin-Hui Fang
Department of Mathematics, Nanjing University of Information Science & Technology, Nanjing

Two infinite sequences A and B of non-negative integers are called infinite additive complements if their sum contains all sufficiently large integers. In this talk, we will present a review of our results (joint with Prof. Yong-Gao Chen) on additive complements, including our recent result on additive complements of the squares. Furthermore, some open conjectures are also posed.

Consecutive primes and Beatty sequences

Zhen-Yu Guo(Victor Z. GUO)
School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an

Beatty sequences are generalized arithmetic progressions which have been studied intensively in recent years. Thanks to the work of Vinogradov, it is known that every Beatty sequence S contains infinitely many prime numbers. For a given pair of Beatty sequences S and T, it is natural to wonder whether there are infinitely many primes in S for which the next larger prime lies in T. In this talk, I will show that this is indeed the case if one assumes a certain strong form of the Hardy-Littlewood conjectures. This is recent joint work with William Banks.