Surface Parameterization via Aligning Optimal Local Flattening

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【OVERVIEW】

◆ Algorithm
  ■ Optimal local flattening
  ■ Global alignment

◆ Contribution
  ✓ Geometry structure preserving
  ✓ Free boundary
  ✓ Dealing with holes
  ✓ Linear constraints
  ✓ Fast and efficient

【ALGORITHM】

◆ Optimal Local Flattening
  ■ Local geodesic polar map [Welch and Witkin 1994]
    \[
    \| x_i - x_i^0 \| = q_i - q_i^0, \quad \beta_j = 2\pi x_j / \sum \alpha_i, \quad j = 1, \ldots, k.
    \]

◆ Global Alignment
  ■ Local distortion measure (LDM)
    \[
    e_i^0 = t_i - (t_i + \mu_i), \quad i = 1, \ldots, N.
    \]

  ■ Global distortion measure (GDM)
    \[
    \text{GDM} = \sum \alpha_i \| E_i \|_F^2 = \sum \alpha_i \| T_i(1-eI) - U_iQ_i \|_F^2.
    \]

◆ Solution
  ■ Minimization with normalization constraint
    \[
    \min \{ \text{GDM} \} = \min \| TSW \|_F^2, \quad TT^T = I.
    \]

    The optimal \( T \) is given by the two eigenvectors of the matrix \( B \) corresponding to the 2nd and 3rd smallest eigenvalues, where
    \[
    B = SWW^T S^T.
    \]

△ Texture Mapping with Feature Constraints
  ■ Minimization with linear constraints
    \[
    \min \{ \text{GDM} \} = \min \| TSW \|_F^2, \quad TT^T = I.
    \]

    \[
    L_{ij} = \left\{ \begin{array}{ll}
    \mu, & j = i, 1 \leq k \leq s, 1 \leq j \leq N, \\
    0, & \text{else},
    \end{array} \right.
    \]

    \[
    C = \mu [c_1, \ldots, c_N].
    \]

    \[
    T \in \mathbb{R}^{N \times N} \text{ s.t. } [SW, L] = [0, C].
    \]

【EXPERIMENTAL RESULTS】

△ Example of parameterizing irregularly sampled model

△ Comparison with Floater’s method [Floater 1997]

△ Comparison with MDS method [Zigelman et al. 2002]