Pricing Currency Options Under Two-Factor Markov-modulated Stochastic Volatility Models

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Abstract

We investigate the valuation of currency options when the dynamics of the spot FX rates is governed by a two-factor Markov-modulated stochastic volatility model, with the first stochastic volatility component driven by a Ornstein-Uhlenbeck mean-reverting process and the second independent stochastic volatility component driven by a continuous-time Markov chain process. The states of the continuous-time Markov chain process can be interpreted as the states of the economy. In the two-factor stochastic volatility model, the short-term fluctuation of the volatility is modelled by the Ornstein-Uhlenbeck mean-reverting process while the long-term variation of the volatility over time due to the transitions of states of the economy is driven by the continuous-time Markov chain process. We employ the regime-switching Esscher transform to determine a martingale pricing measure for valuation of the currency options under the incomplete market described by the stochastic volatility model. We consider the valuation of the European-style and American-style currency options. Both the probabilistic approach and partial differential equation approach for option valuation will be considered. In the case of American options, we formulate the option pricing problem as a free-boundary problem using the partial differential equation approach. We derive the decomposition result for the American option price into the sum of its European counterpart and the early exercise premium by using a probabilistic approach.