1 The Assignment

I. Write a matlab program to reproduce the six plots in Figure 1 of the notes.

II. Write a C++ package to implement the multigrid solver discussed in the notes for the one-dimensional Poisson equation

\[ u''(x) = f(x) \]  

on \( \Omega = [0, 1] \) with the Dirichlet boundary condition \( u(0) = u_0 \) and \( u(1) = u_1 \). Your package must give the user the following options:

(a) boundary conditions: homogeneous and nonhomogeneous;
(b) restriction operators: full-weighting and injection \( (v^h_j = v^h_{2j}) \);
(c) interpolation operators: linear and quadratic;
(d) cycles: V-cycle and full multigrid cycle;
(e) stopping criteria: the number of maximum iterations and the relative accuracy \( \epsilon \) of the solution;
(f) the initial guess.

As for the bottom solver, you can either implement a Gaussian elimination in your package or use the one in the BLAS or LAPACK package.

III. For the function

\[ u(x) = \exp(\sin(x)) \]  

derive the corresponding \( f(x) \) and the boundary conditions. For \( \epsilon = 10^{-8} \) and the zero-vector initial guess, test your multigrid solver for all combinations of (b,c,d) in II on grids with \( n = 128, 256, 512, 1024 \), report the residual and the reduction rate of the residuals for each V-cycle. Report the maximum norm of the error vector and the corresponding convergence rates on the four grids.

IV. Gradually reduce \( \epsilon \) towards \( 2.2 \times 10^{-16} \), under which critical value of \( \epsilon \) does your program fail to achieve the preset accuracy? Why?

V. Design a test with homogeneous boundary conditions and repeat III.

VI. Write a GNU makefile under your root directory so that the command “make run” would trigger the compilation of your source code, the production of the executable, and the running of your tests.

Problem I weighs 10 points and the other problems weigh 90 points.

2 How to submit

You should send your documentation (if there is any) and your matlab and C++ programs in a single gzipped tar ball (format: YourName_Homework1.zip) to my email Qinghai@zju.edu.cn.