1. 解：
   \[ EX_n = E(A \cos \lambda n + B \sin \lambda n) = \cos \lambda n \cdot EA + \sin \lambda n \cdot EB = 0 + 0 = 0; \]
   \[ \text{Cov}(X_n, X_m) = E[(X_n - EX_n)(X_m - EX_m)] = E(X_n X_m) \]
   \[ = E(A^2 \cos \lambda n \cdot \cos \lambda m + AB(\cos \lambda n \cdot \sin \lambda m + \sin \lambda n \cdot \cos \lambda m) + B^2 \sin \lambda n \cdot \sin \lambda m] \]
   \[ = E(A^2 \cdot \cos \lambda n \cdot \cos \lambda m + E(AB)(\cos \lambda n \cdot \sin \lambda m + \sin \lambda n \cdot \cos \lambda m) + E(B^2) \cdot \sin \lambda n \cdot \sin \lambda m \]
   \[ = \cos \lambda n \cdot \cos \lambda m + \sin \lambda n \cdot \sin \lambda m \]
   \[ = \cos(\lambda n - \lambda m). \]

2. 解：
   \[ EX_n = 0; \]
   \[ \text{Cov}(X_n, X_m) = E[(X_n - EX_n)(X_m - EX_m)] = E(X_n X_m) \]
   \[ = \sum_{i=0}^{r} \sum_{j=0}^{r} \alpha_i \alpha_j E(Z_{n-i} Z_{m-j}) = \begin{cases} \sum_{i=0}^{r} \alpha_i \alpha_{r-i}, & \text{若}|n-m| < r + 1; \\ 0, & \text{其它} \end{cases} \]

3. 证明：
   \[ X_n = \alpha X_{n-1} + Z_n \]
   \[ = \alpha(\alpha X_{n-2} + Z_{n-1}) + Z_n \]
   \[ = \alpha^2 X_{n-2} + \alpha Z_{n-1} + Z_n \]
   \[ = \ldots \]
   \[ = \sum_{i=0}^{\infty} \alpha^i Z_{n-i} \]

不妨假设 \( m \geq 0 \).
   \[ \text{Cov}(X_n, X_{n+m}) = \text{Cov}(\sum_{i=0}^{\infty} \alpha^i Z_{n-i}, \sum_{j=0}^{\infty} \alpha^j Z_{m+n-j}) \]
   \[ = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \text{Cov}(Z_{n-i}, Z_{m+n-j}) \cdot \alpha^i \cdot \alpha^j \]

令 \( n - i = n + m - j, \Rightarrow j = m + i \geq 0 \)

所以
   \[ \text{Cov}(X_n, X_{n+m}) = \sum_{i=0}^{\infty} \alpha^i \cdot \alpha^{m+i} \cdot 1 = \alpha^m \sum_{i=0}^{\infty} (\alpha^2)^i = \frac{\alpha^m}{1 - \alpha^2}. \]

若 \( m \leq 0 \), 则
   \[ \text{Cov}(X_n, X_{n+m}) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \text{Cov}(Z_{n-i}, Z_{m+n-j}) \cdot \alpha^i \cdot \alpha^j \]

令 \( n - i = n + m - j, \Rightarrow i = j - m \geq 0 \)

所以
   \[ \text{Cov}(X_n, X_{n+m}) = \sum_{j=0}^{\infty} \alpha^{j-m} \alpha^j \cdot 1 = \alpha^{-m} \sum_{j=0}^{\infty} (\alpha^2)^j = \frac{\alpha^{-m}}{1 - \alpha^2} \]

综上，
   \[ \text{Cov}(X_n, X_{n+m}) = \frac{\alpha^{|m|}}{1 - \alpha^2}. \]
4. 解：引理：随机向量服从多元正太分布的充要条件是它的各分量的任意线性组合服从正太分布。

任给\(t_1, t_2, \ldots, t_n \in \mathbb{R}\)，由上述引理可知\((X(t_1), X(t_2), \ldots, X(t_n))\)服从多元正太分布。

下面计算上述正太分布的均值向量\(\mu\)及协方差矩阵\(\Sigma\)：

由于\(EX(t) = E(Y \cos \theta t + Z \sin \theta t) = EY \cdot \cos \theta t + EZ \cdot \sin \theta t = 0\)，

\[
\text{Cov}(X(t_i), X(t_j)) = \cos((t_i - t_j) \theta),
\]

可得：\(\mu = [0, 0, \ldots, 0]^T\)，\(\Sigma = [\cos((t_i - t_j) \theta)]_{ij}, 1 \leq i, j \leq k\)。

进而，随机向量\((X(t_1), X(t_2), \ldots, X(t_n))\)的密度函数为：

\[
f_{(X(t_1), X(t_2), \ldots, X(t_n))}(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu)}, \text{其中} \ X = (x_1, x_2, \ldots, x_n).\]